Family unification in five and six dimensions

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In family unification models, all three families of quarks and leptons are grouped together into an irreducible representation of a simple gauge group, thus unifying the standard model gauge symmetries and a gauged family symmetry. Large orthogonal groups, and the exceptional groups E_7 and E_8, have been much studied for family unification. The main theoretical difficulty of family unification is the existence of mirror families at the weak scale. It is shown here that family unification without mirror families can be realized in simple five-dimensional and six-dimensional orbifold models similar to those recently proposed for $SU(5)$ and $SO(10)$ grand unification. It is noted that a family unification group that survived to near the weak scale and whose coupling extrapolated to high scales unified with those of the standard model would be evidence, accessible in principle at low energy, of the existence of small (Planckian or GUT-scale) extra dimensions.

I. INTRODUCTION

A. Family unification

It has long been known that theories with extra dimensions afford ways of breaking gauge symmetries unavailable in 4D theories [1]. Recently, such mechanisms have been applied to the problem of constructing realistic grand unified models [2,3]. In particular, it has been shown that orbifold compactification of one or two extra dimensions allows symmetries to be broken in such a way as to resolve in an apparently simple fashion two of the thorniest problems of 4D grand unification, namely, the doublet-triplet splitting problem and the problem of dimension-5 proton decay operators. In this paper we show that the same ideas can be applied to a problem almost equally old, that of "family unification" [4–11].

The idea of family unification is an extension of the idea of grand unification. Grand unification has two aspects: gauge unification and quark-lepton unification. Gauge unification is the unification of all the standard model gauge groups within a simple group, such as $SU(5)$, $SO(10)$, or E_6. Quark-lepton unification is the idea that quarks and leptons are put together into irreducible multiplets of that simple group. There is complete quark-lepton unification if all the fermions of one family are contained in a single irreducible representation. This is possible in $SO(10)$, E_6, and larger groups. The idea of family unification has the same two aspects. It involves, first, the existence of a family gauge group that is unified with the standard model gauge group into a simple group [4], and, second, the existence of a single irreducible representation of that group that contains the quarks and leptons of *all three* known families. For example, if there is a family group $SU(3)$, it can be unified with the standard model group within E_8, since $E_8 \supset E_6 \times SU(3) \supset SU(3) \times SU(2) \times U(1) \times SU(3)$. At the same time, in E_8 all three families can be unified into one irreducible representation, the **248**. The groups that have been most studied from the point of view of family unification are $O(14)$ [7], $SO(16)$ [8], $SO(18)$ [9], E_7 [10], and E_8 [11].

B. The problem of mirror families

The central problem of family unification is that the ordinary $V-A$ families must be accompanied by an equal number of $V+A$ or "mirror" families. For example, the **248** of E_8 decomposes under $E_6 \times SU(3)$ into

$$\mathbf{248} \rightarrow (\mathbf{78},\mathbf{1}) + (\mathbf{1},\mathbf{8}) + (\mathbf{27},\mathbf{3}) + (\overline{\mathbf{27}},\overline{\mathbf{3}}). \tag{1}$$

Together with three ordinary families, $(\mathbf{27},\mathbf{3})$, one finds three mirror families, $(\overline{\mathbf{27}},\overline{\mathbf{3}})$. Similarly, for the orthogonal groups, a spinor of $SO(10+2n)$ decomposes under the $SO(10) \times SO(2n)$ subgroup as

$$2^{(4+n)} \rightarrow (\mathbf{16},2^{(n-1)}) + (\overline{\mathbf{16}},2^{(n-1)'}). \tag{2}$$

(For n odd the $2^{(n-1)}$ and $2^{(n-1)'}$ are conjugate to each other, whereas for n even they are each self-conjugate. For a review of spinor representations of orthogonal groups see [5].) One sees that for every **16** there is a $\overline{\mathbf{16}}$.

The fact that family unification generally gives equal numbers of families and mirror families presents the problem of explaining why no mirror families have been observed. The obvious solution would be to give large mass in some way to the mirror families. However, this turns out to be not so easy. While mirror families could have large $SU(2)_L \times U(1)_Y$-invariant mass terms that couple them to ordinary families, such terms give mass to equal numbers of families and mirror families and so do not resolve the problem. The only way to give the mirror families large masses without giving large mass to the ordinary families is with $SU(2)_L \times U(1)_Y$-breaking mass terms. Such masses would have to be at or below the weak-interaction scale, meaning that the mirror families should be observable through radiative effects in precision tests of the electroweak theory. Recent analyses [12,13] conclude that some additional families or mirror families are allowed, with Ref. [12] finding a chi-squared minimum for the number of extra families to be somewhere between one and two. Adler argues that even

three mirror families may not be completely excluded by present data [14]. However, even if mirror families with masses in the hundred GeV range are consistent with data, explaining in a natural way why their masses are much larger than the observed quarks and leptons will be a theoretical challenge. No simple and plausible mechanism which attempts this is known.

Given that it is quite difficult to hide mirror families at the weak scale the question arises whether in a model with family unification the mirror families might be banished altogether from the low-energy spectrum. One interesting possibility is that the mirror families are confined at a high scale [6,5,15]. Suppose, for example, that the family unification group is $SO(18)$ and breaks down to $SO(10) \times SO(8)$ at a scale M_G. The quarks and leptons are then in the representations $(\mathbf{16}, \mathbf{8}) + (\overline{\mathbf{16}}, \mathbf{8}')$. [See Eq. (2).] If at a scale $M_{fam} \gg M_W$ the family group $SO(8)$ breaks to some confining group H under which the $\mathbf{8}'$ decomposes into only non-singlets, then all of the mirror families are confined. If, on the other hand, $\mathbf{8}$ contains some singlets in its decomposition under H, then some ordinary families will be unconfined and appear in the low-energy spectrum. For instance, there is an $SO(5)$ subgroup of $SO(8)$ under which $\mathbf{8}' \rightarrow \mathbf{4} + \overline{\mathbf{4}}$ but $\mathbf{8} \rightarrow \mathbf{5} + \mathbf{1} + \mathbf{1} + \mathbf{1}$, so that if $SO(5)$ confined three families would remain light [6,5]. Unfortunately, it is doubtful that there exist models of family unification in four-dimensions in which the gauge coupling of H is asymptotically free and thus able to confine the mirror families at some dynamically generated high scale. [The beta function for the $SO(5)$ gauge coupling in the example given above is $dg_5 / d \ln \mu = +\{(62/3)/(16\pi^2)\} g_5^3$ in the non-supersymmetric case, with a larger positive value if there is supersymmetry.]

It would seem, therefore, that in four-dimensional theories mirror families cannot be banished from the low-energy theory. However, it can happen in theories with extra dimensions [16,17,19]. For example, as was shown in Ref. [16], in Calabi-Yau compactification of heterotic string theory, the E_8 of the observable sector can be broken down to E_6 below the compactification scale with a chiral low-energy spectrum of quarks and leptons. In Ref. [17] it was shown that orbifold compactification from ten to four dimensions can leave an $E_6 \times SU(3)$ subgroup of E_8 unbroken, with a chiral low energy spectrum which contains some families in multiplets of the $SU(3)$. (In the particular example of that paper there were thirty-six families [in $3 \times (\mathbf{27}, \mathbf{3}) + 27 \times (\mathbf{27}, \mathbf{1})$] and no mirror families. For early string theory models with three families, see Ref. [18].) Such string theory models have family unification in the sense that all the families of quarks and leptons originate from one E_8 multiplet of the ten-dimensional theory.

In this paper we show that family unification can be achieved in simple 5D and 6D orbifold models quite similar in spirit to those of [2,3]. These are "bottom up" models, like those of [2,3], and no attempt is made to derive them from superstring theory. In the next section, we present simple illustrative models based on both orthogonal groups and E_8. Before we turn to higher dimensions, however, it may be helpful to briefly review some facts about the family unification groups that have been studied in the literature.

C. Family unification groups

First let us look at the smallest group that has been used for family unification, $O(14)$. We see from Eq. (1) that a spinor of $SO(14)$ decomposes under $SO(10) \times SO(4)$ as $\mathbf{64} \rightarrow (\mathbf{16}, \mathbf{2}) + (\overline{\mathbf{16}}, \mathbf{2}')$. The conjugate spinor decomposes as $\overline{\mathbf{64}} \rightarrow (\mathbf{16}, \mathbf{2}') + (\overline{\mathbf{16}}, \mathbf{2})$. Thus, a single spinor of $SO(14)$ can only accomodate two families and their mirrors. However, if one takes the group to be $O(14)$ [that is $SO(14)$ together with the parity that transforms the $\mathbf{64}$ and $\overline{\mathbf{64}}$ into each other] then the irreducible spinor is $\mathbf{128}(=\mathbf{64} + \overline{\mathbf{64}})$, which can accommodate four families. Models of family unification based on $O(14)$ have been constructed [7].

The group $SO(16)$ is large enough for family unification, since $\mathbf{128} \rightarrow (\mathbf{16}, \mathbf{4}) + (\overline{\mathbf{16}}, \overline{\mathbf{4}})$ [and $\mathbf{128}' \rightarrow (\mathbf{16}, \overline{\mathbf{4}}) + (\overline{\mathbf{16}}, \mathbf{4})$]. However, its spinors are real, i.e. self-conjugate, so that in the absence of any other symmetry there would be nothing to prevent a mass term of the form $\mathbf{128} \times \mathbf{128}$, which would naturally make all the quarks and leptons superheavy. [The same objection applies to $O(14)$.] For this reason most authors have concentrated on the group $SO(18)$ [9], which is the smallest orthogonal group whose spinors are complex and large enough to contain at least three families. However, it is possible to avoid the self-mass problem in $SO(16)$ by assuming that the $\mathbf{128}$ of quarks and leptons is charged under some other group, say a $U(1)$. Interesting $SO(16)$ models using this idea have been constructed [8].

The group E_7 has an adjoint that under the $SU(5) \times SU(3)$ subgroup decomposes as $\mathbf{133} \rightarrow (\mathbf{24}, \mathbf{1}) + (\mathbf{1}, \mathbf{8}) + (\mathbf{5}, \mathbf{1}) + (\overline{\mathbf{5}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}) + (\mathbf{10}, \overline{\mathbf{3}}) + (\overline{\mathbf{5}}, \mathbf{3}) + (\overline{\mathbf{10}}, \mathbf{3}) + (\mathbf{5}, \overline{\mathbf{3}})$. As with the other groups we have discussed, this has an equal number of families and mirror families. But it also has the interesting peculiarity that some of the fermions in a family transform as $\mathbf{3}$ of the $SU(3)$ family group, while others transform as $\overline{\mathbf{3}}$. It should be noted also that the self-mass problem exists for both E_7 and E_8 [as it does for $O(14)$ and $SO(16)$] since both the $\mathbf{133}$ of E_7 and the $\mathbf{248}$ of E_8 are real representations.

II. FAMILY UNIFICATION WITHOUT THE MIRROR FAMILIES

We start with a simple $SO(16)$ model of family unification in five dimensions in which the low-energy four-dimensional theory has three light families in a triplet of $SU(3)$ and no mirror families.

Following in the footsteps of [2], we consider a five-dimensional theory with the fifth dimension (y) compactified on an $S^1/(Z_2 \times Z_2')$ orbifold. The circumference of S^1 is $2\pi R$. Z_2 reflects $y \rightarrow -y$, and Z_2' reflects $y' \rightarrow -y'$, where $y' = y + \pi R/2$. (See Fig. 1.) The orbifold $S^1/(Z_2 \times Z_2')$ can be taken to be the interval $-\pi R/2 \leqslant y \leqslant 0$. The point $y=0$, which we will call O, is a fixed point of Z_2, while the point $y = -\pi R/2$ (or $y' = 0$), which we will call O', is a fixed point of Z_2'. There are branes at the points O and O'. It is
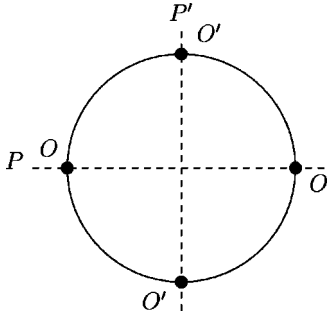
FIG. 1. The fifth dimension compactified on an $S^1/(Z_2 \times Z_2')$ orbifold.

assumed that in the five-dimensional bulk there is an $N=1$ supersymmetric theory with gauge group $SO(16)$ and fields consisting of a vector multiplet in the adjoint (**120**) and a hypermultiplet in the spinor (**128**).

The $N=1$ supersymmetry in five dimensions would give $N=2$ supersymmetry in four dimensions, but the orbifold compactification will break this down to $N=1$. The vector multiplet of the bulk theory splits into a vector multiplet V^a and a chiral multiplet Σ^a of the unbroken 4D $N=1$ supersymmetry. Under the Z_2 symmetry V^a is taken to have parity $+$ and Σ^a to have parity $-$. Similarly, the hypermultiplet splits into two chiral multiplets Φ and Φ^c having opposite Z_2 parity. The Z_2' parities are assigned in the following way. The $SO(16)$ adjoint when decomposed under $SO(10) \times SO(6)$ gives **120**$\rightarrow$(**45,1**)$+$(**1,15**)$+$(**10,6**). We assign Z_2' parity $+$ to the components of the 5D vector multiplet in the (**45,1**)$+$(**1,15**), and Z_2' parity $-$ to those in the (**10,6**). This assignment is consistent with the group algebra of $SO(16)$. The spinor of $SO(16)$ decomposes as **128**$\rightarrow$(**16,4**) $+$($\overline{\bf 16},\overline{\bf 4}$). Under Z_2' we take the components of the 5D hypermultiplet that are in (**16,4**) to have parity $+$ while those in ($\overline{\bf 16},\overline{\bf 4}$) have parity $-$. The complete $Z_2 \times Z_2'$ assignments are shown in Table I.

As explained in many papers [2], only those fields that have $Z_2 \times Z_2'$ parity $(+,+)$ can have among their Kaluza-Klein modes one that is constant in the fifth dimension, i.e. a zero mode corresponding to a light field in the four-dimensional effective theory. Fields that have $(+,-)$, $(-,+)$, or $(-,-)$ parities must vanish at the fixed points O, O', or both, and therefore have only higher Kaluza-Klein modes, which are superheavy from the 4D point of view. In this model the four-dimensional effective theory will have $N=1$ supersymmetry, gauge group $SO(10) \times SO(6)$, and light fields coming from the bulk matter consisting of vector multiplets in (**45,1**) and (**1,15**) and a chiral multiplet in (**16,4**). This cannot be the complete effective low-energy theory since it has an $SO(6)^3$ gauge anomaly. One must take

into account also fields that "live" on the branes at O and O'.

On the brane at O there is the full $SO(16)$ gauge symmetry, but on the brane at O' there is only an $SO(10) \times SO(6)$. It is not that $SO(16)$ is spontaneously broken on the O' brane, but rather that there never was any gauge symmetry on O' except $SO(10) \times SO(6)$. (At O' quantities that are odd under Z_2' have to vanish, including the gauge parameters $\xi^a(x^\mu, y)$ for those generators λ^a in the coset $SO(16)/[SO(10) \times SO(6)]$.) Consequently, in the 4D theory on the brane at O' only complete multiplets of $SO(10) \times SO(6)$ but not $SO(16)$ need appear. Let us suppose then that on the O' brane one has quark and lepton chiral multiplets in ($\overline{\bf 16},{\bf 1}$)$+2\times$(**1,10**) and Higgs chiral multiplets in (**1,4**)$+$(**1**,$\overline{\bf 4}$). (We will denote the Higgs multiplets henceforth with subscript "H.") The low-energy theory then has altogether the following chiral multiplets: (**16,4**) $+$($\overline{\bf 16},{\bf 1}$)$+2\times$(**1,10**)$+$(**1,4**)$+$(**1**,$\overline{\bf 4}$). This set has no gauge anomalies. [The $SO(6)^3$ anomaly of a **10** is -8 times that of a **4**, so the total $SO(6)^3$ anomaly of the above set is $16(1) +2(-8)=0$.]

We assume that the (**1,4**)$_H$$+$(**1**,$\overline{\bf 4}$)$_H$ obtain vacuum expectation values at some scale $M_{family} \gg M_W$ that breaks $SO(6)$ $[=SU(4)]$ down to $SU(3)$. Then a term of the form (**16,4**)($\overline{\bf 16},{\bf 1}$)(**1**,$\overline{\bf 4}$)$_H$ on the O' brane will give mass to the ($\overline{\bf 16},{\bf 1}$) $V+A$ family and one of the four $V-A$ families to leave three light families in a (**16,3**) of $SO(10) \times SU(3)$. Note that the scale M_{family} can be anything from the weak scale up to the compactification scale.

There still remain the questions of how $SO(10)$ breaks down to the standard model group in a satisfactory way, how the electroweak symmetry is broken, how realistic masses arise for the quarks and leptons, and how the $SU(3)$ family symmetry breaks. These breakings can all be accomplished through the ordinary Higgs mechanism by Higgs multiplets living on the O' brane in suitable representations of $SO(10) \times SO(6)$. We know, for example, that breaking of $SO(10)$ down to the standard model can be achieved in 4D supersymmetric (SUSY) ground unified theories (GUTs) in such a way as to have natural doublet-triplet splitting [20] and also suppression of dimension-5 proton decay operators (i.e. those coming from colored Higgsino exchange) [21]. It would be interesting to attempt to use orbifold compactification to break $SO(10)$ down to the standard model as well, as in [3]. However, we do not pursue that more ambitious goal in this paper.

What we have shown in this simple example is that models can be constructed in which the three light families come from a single irreducible representation of a simple group— here a hypermultiplet in the bulk $SO(16)$ theory, and in

TABLE I. (Z_2, Z_2') parity assignments for the vector and hypermultiplets in $SO(16)$.

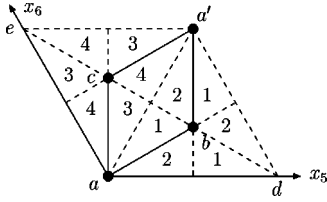|	$V_{(\mathbf{45,1})}$	$V_{(\mathbf{1,15})}$	$V_{(\mathbf{10,6})}$	$\Sigma_{(\mathbf{45,1})}$	$\Sigma_{(\mathbf{1,15})}$	$\Sigma_{(\mathbf{10,6})}$	$\Phi_{(\mathbf{16,4})}$	$\Phi_{(\overline{\mathbf{16}},\overline{\mathbf{4}})}$	$\Phi^c_{(\mathbf{16,4})}$	$\Phi^c_{(\overline{\mathbf{16}},\overline{\mathbf{4}})}$
Z_2	$+$	$+$	$+$	$-$	$-$	$-$	$+$	$+$	$-$	$-$
Z_2'	$+$	$+$	$-$	$+$	$+$	$-$	$+$	$-$	$-$	$+$

FIG. 2. The two extradimensions x_5 and x_6 compactified on an S^1/Z_3 orbifold. The regions with the same numbers are identified with each other. a, b, and c are fixed points of Z_3, and the parallelogram $aba'c$ is the fundamental region.

which that simple group comprises both the standard model gauge symmetries and a family gauge symmetry that survives in the four-dimensional theory, perhaps even down to low energy. It is clear that one may construct $SO(18)$ models in a similar way. However, the analogous orbifold breaking to $SO(10)\times SO(8)$ would give *eight* families coming from the hypermultiplet in the bulk. One would therefore have to introduce *five* mirror families on the O' brane, and a correspondingly large number of Higgs bosons on the brane to mate the mirror families with the ordinary ones. It would be interesting to know if a more economical and compelling way to obtain three light families from $SO(18)$ using orbifold compactifications could be found.

We see, then, that the orthogonal groups can accomodate three light families, but they do not naturally prefer three light families. If we wish to explain the fact that there are three light families, the group E_8 seems more promising, given that $E_6\times SU(3)$ is a maximal subgroup and that the fundamental representation of E_8 contains a $(\mathbf{27,3})$ under that subgroup, as shown in Eq. (1).

It is easy to see that orbifolds with Z_2 symmetries as we have been considering up to now are not adequate to split the $(\overline{\mathbf{27}},\overline{\mathbf{3}})$ mirror families from the $(\mathbf{27,3})$ families in the $\mathbf{248}$. For a Z_2 parity to accomplish this, the $(\overline{\mathbf{27}},\overline{\mathbf{3}})$ and $(\mathbf{27,3})$ would have to have opposite parity. Suppose, then, we imagine the parity of $(\mathbf{27,3})$ to be $+$ and that of $(\overline{\mathbf{27}},\overline{\mathbf{3}})$ to be $-$. This would be inconsistent with the group algebra of E_8. The 248 generators of E_8 fall into $(\mathbf{78,1})+(\mathbf{1,8})+(\mathbf{27,3})$ $+(\overline{\mathbf{27}},\overline{\mathbf{3}})$. The commutator of two $(\mathbf{27,3})$ generators gives a $(\overline{\mathbf{27}},\overline{\mathbf{3}})$ generator. Similarly, the commutator of two $(\overline{\mathbf{27}},\overline{\mathbf{3}})$ generators gives a $(\mathbf{27,3})$ generator. Consequently the gauge fields in $(\mathbf{27,3})$ and $(\overline{\mathbf{27}},\overline{\mathbf{3}})$ must both have $+$ parity. That implies that the families and mirror families have the same parity.

In order to eliminate the mirror families from the 4D theory in E_8 there must be an orbifold with at least a Z_3 symmetry. The point is that under a Z_3 the generators of E_8 can transform as follows consistently with the group algebra:

$$\lambda_{(78,1)}\rightarrow\lambda_{(78,1)}$$

$$\lambda_{(1,8)}\rightarrow\lambda_{(1,8)}$$

$$\lambda_{(27,3)}\rightarrow\omega\lambda_{(27,3)}$$

$$\lambda_{(\overline{27},\overline{3})}\rightarrow\omega^2\lambda_{(\overline{27},\overline{3})}, \tag{3}$$

where $\omega=e^{2\pi i/3}$. Such a Z_3 can be used to split the families from the mirror families in a $\mathbf{248}$, as will be seen.

Consider a six-dimensional theory in which the extra two dimensions (x^5 and x^6) are compactified on an orbifold with a Z_3 symmetry. The construction of such an orbifold is shown in Fig. 2. One starts with a torus T^2 defined by identifying the points $z\equiv x^5+ix^6$ with the points $z+1$ and $z+\omega$. This torus is shown as the large parallelogram in Fig. 2. Under a rotation by $2\pi/3$ (i.e. $z\rightarrow\omega z$) one sees that the regions labeled 1 get mapped into other regions labeled 1, and that similarly 2 gets mapped into 2, 3 into 3, and 4 into 4. The T^2/Z_3 orbifold obtained from T^2 by identifying z with ωz thus has as its fundamental region the smaller parallelogram $aba'c$. This orbifold has a Z_3 symmetry, under which a, b, and c are inequivalent fixed points. (The point a' is identified with a). (The orbifold used in [17] is the six-dimensional generalization of this simple two-dimensional orbifold and has 27 fixed points.)

Instead of using one discrete symmetry to break supersymmetry down to $N=1$ and another discrete symmetry to break the gauge group as before, we will use the same Z_3 to break both. In the six-dimensional bulk we take the theory to have $N=1$ supersymmetry and gauge group E_8. (One need not worry about cancellation of gauge anomalies since the E_8 box anomaly in six dimensions vanishes up to "reducible" pieces that can be canceled by the Green-Schwarz mechanism [22].) We take the matter in the bulk to consist of a vector multiplet in the $\mathbf{248}$. A 6D vector multiplet decomposes, under the 4D $N=1$ supersymmetry, into a vector multiplet V^a and a chiral multiplet Σ^a. The index a labels the 248 generators of E_8, which may be classified under the $E_6\times SU(3)$ subgroup as shown in Eq. (1). The fields in V^a are assumed to transform under Z_3 in the same way as the corresponding generators, which is shown in Eq. (3). The fields in Σ^a transform in the same way except with an additional factor of ω^2. The Z_3 transformation properties of the fields are given in Table II.

Only those component fields that transform trivially under Z_3 will have zero modes. Thus, the bulk matter contributes the following fields to the 4D low energy theory: vector multiplets in $(\mathbf{78,1})$ and $(\mathbf{1,8})$ containing the gauge fields and gaugino fields of E_6 and $SU(3)$, and a chiral multiplet in $(\mathbf{27,3})$ containing three families of quarks and leptons. This cannot be the whole story as the 4D $SU(3)^3$ gauge anomaly

TABLE II. Z_3 transformation properties of the vector and chiral multiplets in E_8.

	$V_{(\mathbf{78,1})}$	$V_{(\mathbf{1,8})}$	$V_{(\mathbf{27,3})}$	$V_{(\overline{\mathbf{27}},\overline{\mathbf{3}})}$	$\Sigma_{(\mathbf{78,1})}$	$\Sigma_{(\mathbf{1,8})}$	$\Sigma_{(\mathbf{27,3})}$	$\Sigma_{(\overline{\mathbf{27}},\overline{\mathbf{3}})}$
Z_3	1	1	ω	ω^2	ω^2	ω^2	1	ω

must be canceled. This can be done by chiral multiplets living on one or more of the branes at the fixed points a, b, and c. For example, the $SU(3)^3$ anomaly would be canceled by twenty-seven $(\mathbf{1},\overline{\mathbf{3}})$ or by a single $(\mathbf{1},\overline{\mathbf{10}})$ of $E_6 \times SU(3)$ on one of the branes. It should be noted that on the branes there is only an $E_6 \times SU(3)$ symmetry to start with, so that it is only necessary to have complete multiplets of that group rather than of the bulk gauge group E_8. Again, as in the previous example, we can introduce various (vectorlike) Higgs representations on the branes to break the symmetry down to that of the standard model. Such model-building details are beyond the scope of this paper. But we see no obstacle to constructing a fully realistic model in this way.

III. FAMILY UNIFICATION AS AN INDICATOR OF EXTRA DIMENSIONS

As we have seen, family unification involves the idea that all three known families reside within one irreducible representation of some simple group. This group contains not only the standard model gauge groups, but generally a family gauge group. In many cases this family gauge group contains an $SU(3)$ subgroup [or is an $SU(3)$] under which the three families form a triplet. The family gauge group could be broken at very high scales, either by an ordinary 4D Higgs field or by the orbifold compactification. But it is also quite possible that a local family symmetry, perhaps an entire $SU(3)$, survives down to "low scale," by which we mean here something in the hundred TeV range.

If family unification is realized in nature and the family group survives down to low energy the interesting possibility would exist, at least in principle even if quite difficult in practice, to infer the existence of near-Planckian extra dimensions from experiments done at "low energy."

Let us call the set of representations of $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ that make up one family of left-handed fermions $\mathbf{F}$, and suppose for specificity that the low energy family symmetry is $SU(3)$ [though most of what we will say applies as well to other groups, such as $SO(3)$]. Typically, in models with family unification the known quarks and leptons are in $(\mathbf{F},\mathbf{3})$ of $G_{SM} \times SU(3)_{fam}$. [It is also possible that some of the left-handed fermions of a family are in $\mathbf{3}$ and some in $\overline{\mathbf{3}}$ of $SU(3)_{fam}$, as we saw in the case of family unification in the adjoint of E_7. What we say below applies to such cases as well.]

If $SU(3)_{fam}$ is broken at a scale M_{fam} near enough to the weak scale to be accessible to experiment, then one might be able eventually to do two things: first, to find out what representations of the family group the quarks and leptons are in; and, second, to measure the $SU(3)_{fam}$ gauge coupling g_{fam} and extrapolate it to high energies to see whether it unifies with the standard model couplings. Suppose that this is done and it is found that the quarks and leptons of the standard model are indeed in $(\mathbf{F},\mathbf{3})$, with no light mirror families, and that the four gauge couplings do indeed unify. What would one be able reasonably to infer from this?

First, the evidence for the unification of the four gauge groups (into one simple group or a product of identical simple groups related by a discrete symmetry) would then be compelling. The unification that we now see of the three standard model gauge couplings might be an accident, since it really involves only one non-trivial condition being satisfied. However, the accidental unification of *four* couplings at one point could hardly be dismissed as accidental.

One would therefore conclude that at a high scale some unified group G broke down to $G_{SM} \times SU(3)_{fam}$, and that some anomaly free set of representations of G broke into $(\mathbf{F},\mathbf{3})$ plus some additional multiplets that were vectorlike under G_{SM}. (They would have to be vectorlike under G_{SM} in order not to have been seen at the weak scale.) The question that would then confront theorists is whether such a breaking could take place in the context of a theory with only four spacetime dimensions. The answer seems to be no.

We have seen that groups such as $SO(16)$, $SO(18)$, and E_8 have representations that contain $(\mathbf{F},\mathbf{3})$ of $G_{SM} \times SU(3)$, but these are always accompanied by mirror families, and in four dimensions there does not seem to be any way to push these mirror families to very large scales. The unitary groups, in particular $SU(N)$ with $N \geqslant 8$, also can break down to $G_{SM} \times SU(3)$ giving standard model multiplets that are in triplets of $SU(3)$, but one never gets simply three families plus pieces that are vectorlike under G_{SM}. For example consider $SU(8)$ with quarks and leptons in $\psi^{[\alpha\beta\gamma]} + \psi_{[\alpha\beta]} + \psi_\alpha$. Under the $SU(5) \times SU(3)$ subgroup these do give $(\mathbf{10},\mathbf{3}) + (\overline{\mathbf{5}},\overline{\mathbf{3}})$, but these come together with $2(\overline{\mathbf{10}},\mathbf{1}) + (\mathbf{5},\overline{\mathbf{3}}) + (\overline{\mathbf{5}},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{1},\overline{\mathbf{3}}) + (\mathbf{1},\mathbf{1})$, which are chiral under G_{SM}.

The closest that one seems to be able to come is with the group $SU(5) \times SU(5)$, with the two gauge couplings forced to be equal by a discrete symmetry that interchanges the two $SU(5)$'s. Consider the following anomaly-free set of fermions: $(\mathbf{10},\overline{\mathbf{5}}) + (\overline{\mathbf{5}},\mathbf{10}) + 2(\overline{\mathbf{10}},\mathbf{1}) + 2(\mathbf{1},\overline{\mathbf{10}}) + 7(\mathbf{5},\mathbf{1}) + 7(\mathbf{1},\mathbf{5})$. Under the $SU(5) \times SU(3)$ subgroup this gives $(\mathbf{10},\overline{\mathbf{3}}) + (\overline{\mathbf{5}},\overline{\mathbf{3}})$ together with $2(\overline{\mathbf{5}},\mathbf{3}) + 6(\mathbf{5},\mathbf{1})$ and other pieces that are vectorlike under $SU(5) \times SU(3)$. This indeed gives $(\mathbf{F},\overline{\mathbf{3}})$ plus pieces that are vectorlike under G_{SM}. However, as one can see, those pieces include *six* $\overline{\mathbf{5}} + \mathbf{5}$ of $SU(5)$ that cannot get mass above M_{fam} because they are chiral under $SU(3)_{fam}$. In a supersymmetric theory this would make the couplings blow up below the unification scale.

On the other hand, we have seen that in theories with extra dimensions one can end up with three families in triplets of an $SU(3)$ family gauge symmetry whose coupling unifies at high scales with the standard model gauge couplings, without there being any light mirror families. Such a situation, if it is found, would be a telltale sign at low energy of a theory with extra dimensions at very high scales not directly accessible to experiment.

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